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Radar Cross Sections of Distributed Targets Low Over Water in the Presence of Multipath Reflections

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14. ABSTRACT
The radar cross section (RCS) of 667 reflectors displayed in a 1-m-high array directly above the sea surface is calculated with multipath present and low grazing angles vs range. The RCS is calculated for the array vertical, and tilted 3.38 degrees and 15 degrees from vertical. The results are compared with the RCS that would have been seen in free space. Graphs of the specular reflection coefficient vs sea wave height for 3, 10, and 35 GHz, with grazing angles as parameter, are included. A method to calculate the free space RCS of a target if the radar response vs range is known in a multipath environment is described. If the reflector has a rough surface (maximum peaks $\geq\lambda/4$) for the geometry considered, the RCS for full multipath is fairly constant for ranges above 1 km, and 5 to 10 dB above the RCS without multipath, i.e., free space.

15. SUBJECT TERMS
radar cross sections, distributed targets, multipath low grazing angles, water

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RADAR CROSS SECTIONS OF DISTRIBUTED TARGETS LOW OVER WATER IN THE PRESENCE OF MULTIPATH REFLECTIONS

INTRODUCTION

Recently there was some interest in determining the detection range of small, distributed targets (such as periscopes) low over the ocean in the presence of multipath. Average radar cross sections (RCS) of those objects measured in sea trials routinely exceeded those measured in free space by 5 to 7 dB. In order to explain this, the RCS of a line of equally spaced reflectors suspended quasiperpendicularly low over the ocean is treated in the following calculations.

Theory

Figure 1 shows the four reflection paths from a point source over the ocean. The height of the radar above the water is denoted as H, the height of the point reflector is x, the horizontal distance between radar and target is R, and the reflection coefficient on the water surface is ρ. For horizontal polarization and sea state 0 (i.e., no waves), the reflection coefficient $\rho = -1$. For higher sea states, ρ tends towards zero, more so for shorter electrical wavelengths and higher grazing angles. Figures 2, 3, and 4 show $|\rho|$ as a function of rms waveheight with grazing angle as parameter for 3, 10, and 35 GHz, respectively (adapted from Miller et al.[*]).

The lengths of the return paths are (see Fig.1)

- Direct path $2\,s1$
- First reflected path $s1 + s3 + s2$
- Second reflected path $s2 + s3 + s1$
- Third reflected path $2(s2 + s3)$

The voltage received by the radar is the vector sum v of the voltages from all four paths (the path loss due to the fourth power of range was taken out):

$$\mathrm{v} = \left(e^{j\,4\pi/\lambda\, s1} + 2\rho\, e^{j\,2\pi/\lambda\,(s1+s2+s3)} + \rho^2\, e^{j\,4\pi/\lambda\,(s2+s3)}\right) vf \tag{1}$$

where vf is the voltage that would be created from the target in free space.

$$s1 = \sqrt{(x-H)^2 + R^2} \tag{2}$$

* A.R. Miller, R.M. Brown, and E. Vegh, "New Derivation for the Rough-Surface Reflection Coefficient and for the Distribution of Sea-Wave Elevations," *IEE Proceedings* **131**(H2), April 1984.

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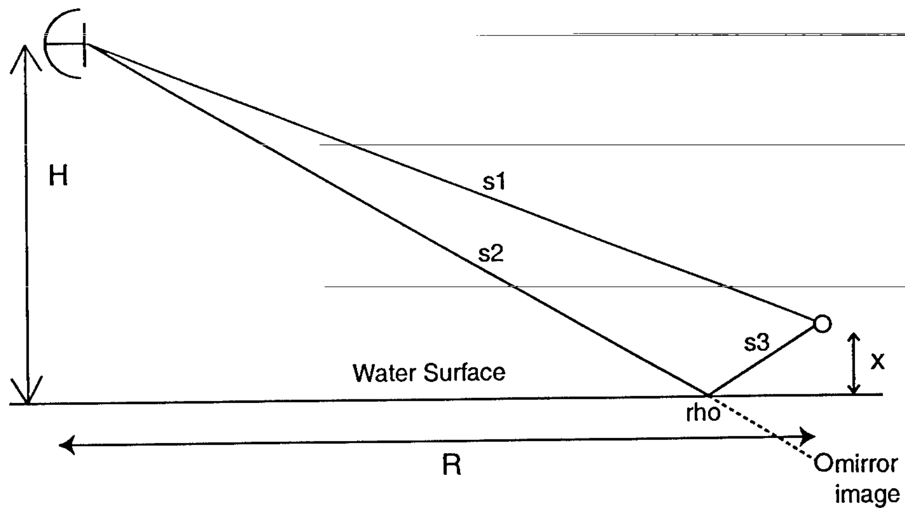

Fig. 1 — Geometry of the multipath scenario with a single reflector

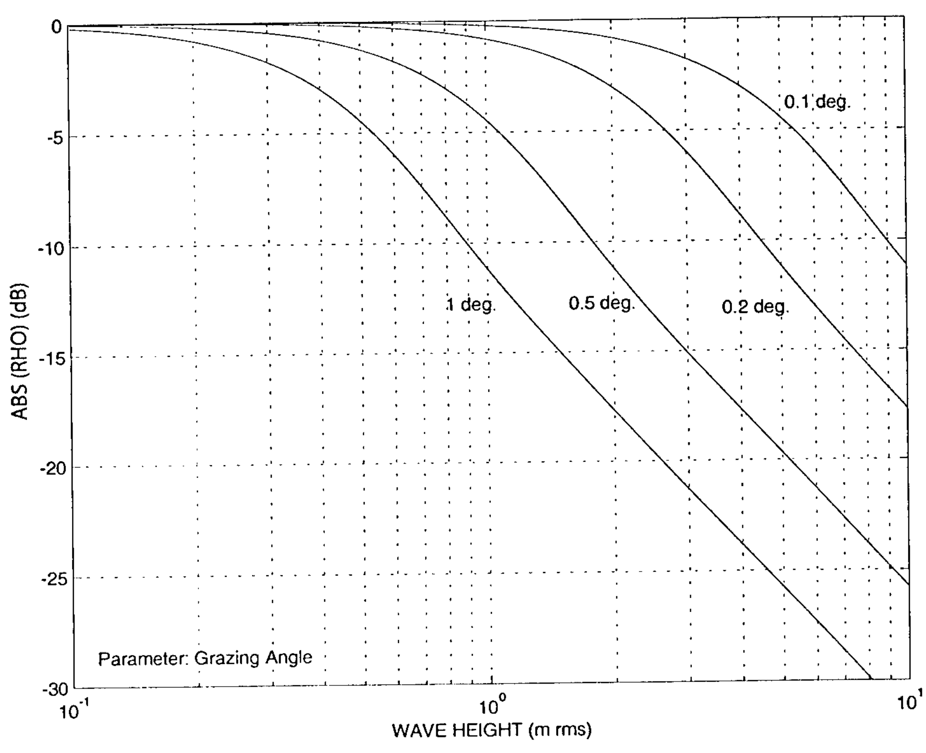

Fig. 2 — Specular reflection coefficient over sea at 3 GHz

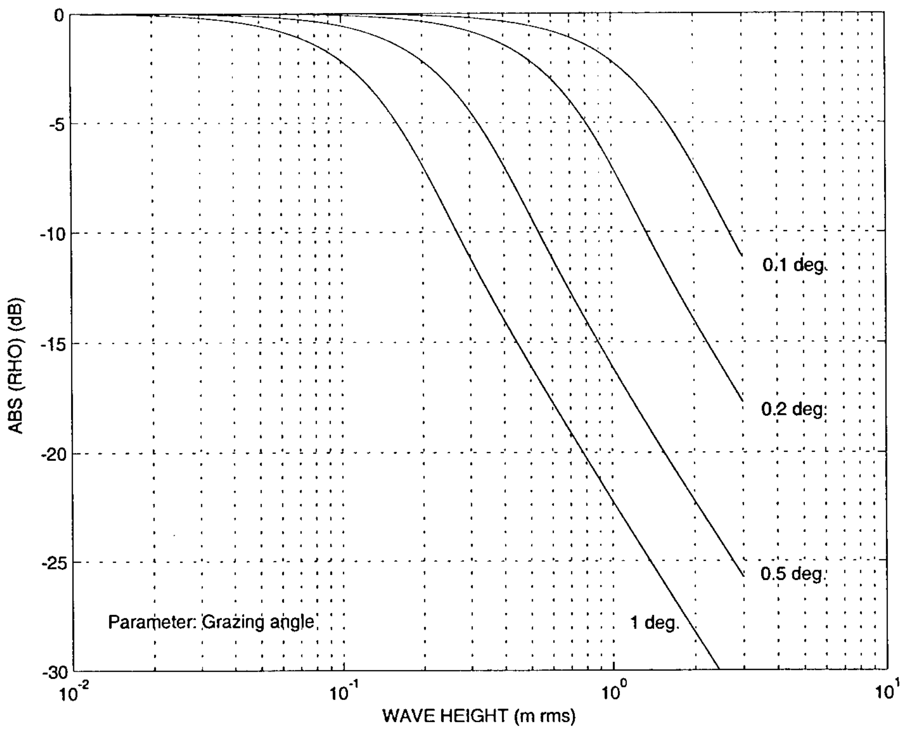

Fig. 3 — Specular reflection coefficient over sea at 10 GHz

For $(x\text{-}H)^2 << R^2$,

$$s1 \approx R + 1/2\,(x\text{-}H)^2\,/R \tag{2a}$$

$$s2 + s3 \approx R + 1/2\,(x\text{+}H)^2\,/R \tag{3}$$

$$s1\text{+}s2\text{+}s3 \approx 2R + (x^2\text{+}H^2)/R \tag{4}$$

Letting

$$a = 2\pi/\lambda\,(x\text{+}H)^2/R \qquad b = 2\pi/\lambda\,(x\text{-}H)^2/R \qquad c = 2\pi/\lambda\,(x^2 + H^2)/R,$$

$$\mathrm{v} = e^{j4\pi R/\lambda}\,(e^{jb} + 2\rho\, e^{jc} + \rho^2 e^{ja})\, vf \tag{1a}$$

$$|\mathrm{v}| = |(e^{jb} + 2\rho\, e^{jc} + \rho^2 e^{ja})|\ |vf| \tag{5}$$

The square of the absolute value of v is obtained by multiplying v by its conjugate complex. It can be shown that

$$|\mathrm{v}|^2 = (\,1 + \rho^2 + 2\rho\cos(\,u\,)\,)^2\ |vf|^2, \quad \text{where } u = 4\pi\, 3\, H\,/\,(\lambda R). \tag{6}$$

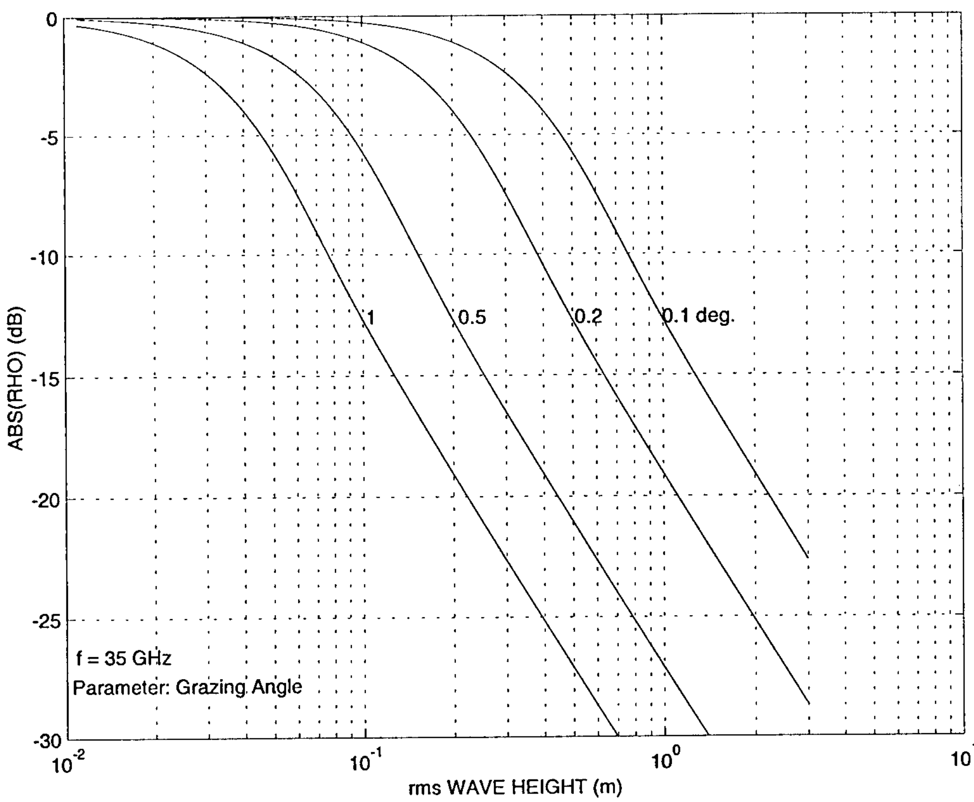

Fig. 4 — Specular reflection coefficient over sea at 35 GHz

Determination of Free Space RCS of a Point Target from the Multipath Maxima and Minima

If the multipath minima and maxima of an encountered point target are known as it moves at constant height above the water surface towards or away from the radar, the following calculations are used to determine its free space RCS.

The maximum occurs when $\rho<0$ and $u = (2k+1)\pi$, where k is an integer. Then, with Eq. (6),

$$|v|^2_{max} = p_{max} = (1 + |\rho|)^4 \; |vf|^2 \tag{7}$$

$$|v|^2_{min} = p_{min} = (1 - |\rho|)^4 \; |vf|^2 \tag{8}$$

Let $w = |v|_{max} / |v|_{min}$ the ratio of the maximum to minimum voltage observed by the radar receiver as the point target moves from a multipath maximum to the subsequent minimum at constant height, then the free space radar cross section of the target σ_f can be determined by the formula

$$\sigma_f = \sigma_{max}\left(1 + 2/\sqrt{w} + 1/w\right)^2 / 16, \tag{9}$$

where

σ_{max} = RCS of point target measured at a multipath maximum

w = $\sqrt{\sigma_{max} / \sigma_{min}}$

σ_{min} = RCS measured at adjacent multipath minimum

The cross sections must be corrected to be referenced to the same range by using the $1/R^4$ law of the radar equation.

Multipath Pattern from a Single Reflector

Figure 5 shows the familiar multipath pattern resulting from a point target moving to or from the radar at constant height of 1 m above the sea level. The free space return is referenced to 0 dB. For clarity, the R^{-4} law of the radar equation was omitted in the result, representing the normalized RCS. The peaks are 12 dB above free space. The reflection coefficient ρ was assumed to be -1, i.e., sea state 0. The frequency was 10 GHz, the height of the radar above sea level was 59 m.

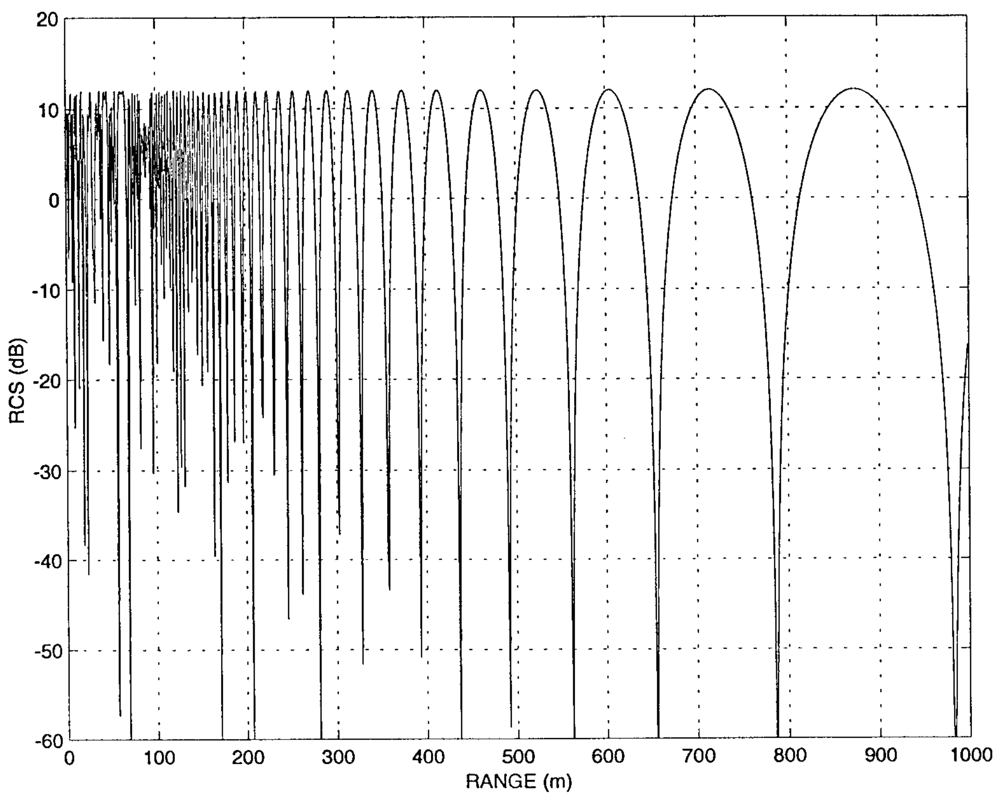

Fig. 5 — RCS with multipath, single reflector at 1 m above sea surface (radar height: 59 m; sea state: 0 (calm); ρ = -1; and f = 10 GHz)

Multipath Pattern of Distributed Targets

The target is assumed to be flat rising from the water surface to a height x_{max}. It is tilted from the perpendicular by an angle α. According to Huygens principle, the target can be represented by many equally spaced reflectors, provided that the spacing is small in comparison to the electrical wavelength (see Fig. 6). Then, using Eq. (5), we can compute the normalized radar return from this target by simply summing the voltages resulting from each reflection point:

$$|v| = \left|\sum_{v}^{nmax} (e^{jb} + 2\rho\, e^{jc} + \rho^2 e^{ja})\right| \quad |vf| \ / \ nmax, \tag{10}$$

with

$$b = 2\pi\,(xv\ + H)2\ /(\lambda\,R) +\ 4\pi\,\mathrm{xv}\,/\lambda\ \ \sin(\alpha) \qquad (10a)$$

The added term in b is taking into account the additional phase shift due to the tilt. The same term is added to c and a.

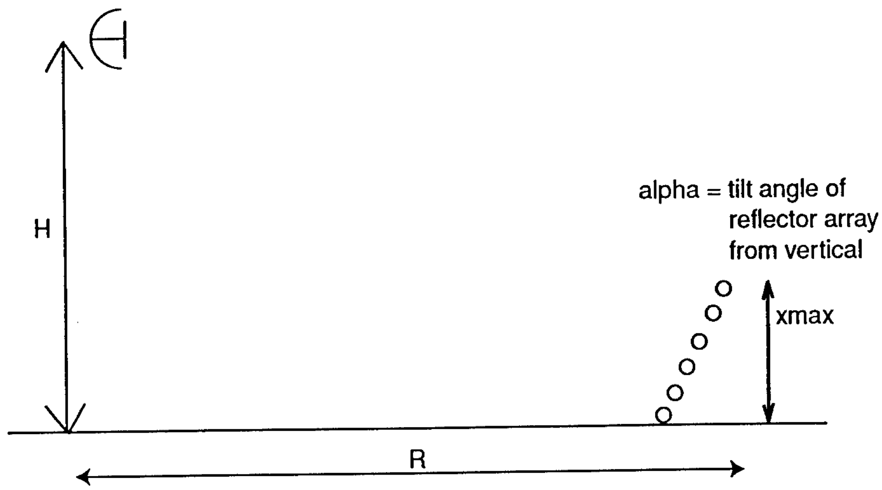

Fig. 6 – Geometry of the reflector array

Equation (10) was used to obtain the following results. Figure 7 shows the pattern resulting from three reflectors spaced perpendicularly above the sea surface at 0, 0.5, and 1-m heights. Some of the minima are filled in, and the maxima have come down from 12 dB to about 6 dB.

Figure 8 shows the same scenario as Fig. 1 but with 667 reflectors spaced equally within zero to 1 m height perpendicularly above sea level. The result may be somewhat unexpected. The multipath pattern has disappeared; the return is independent of range equal to about 6 dB above free space peak return of a linear array.

Figure 9 shows the scenario of Fig. 8 but with no multipath, i.e., free space. This represents the reflection pattern of a linear array, with the aspect angle of the array varying with the range. At a range of 1000 m, the RCS of the particular sidelobe peak is 23 dB below the free space maximum and 29 dB below the return seen with multipath as in Fig. 8. It is clear that multipath plays a decisive role in this case. The sea surface and the reflector act like a dihedral reflector.

Next, we consider what happens when the reflectors are tilted off perpendicular by 3.3 degrees, with no multipath reflections present. At a range of 1000 m we see the peak return of the flat plate, because at that range and 59 m height of the radar, the beam hits the reflector array perpendicularly. Figure 10 shows the result. The maxima and minima are due to the sidelobes of the reflection pattern. When we add multipath, we see the result shown in Fig. 11. The peak at 1000 m range is almost unaffected by multipath reflections, but at ranges below 500 m, the return is increased by up to 20 dB.

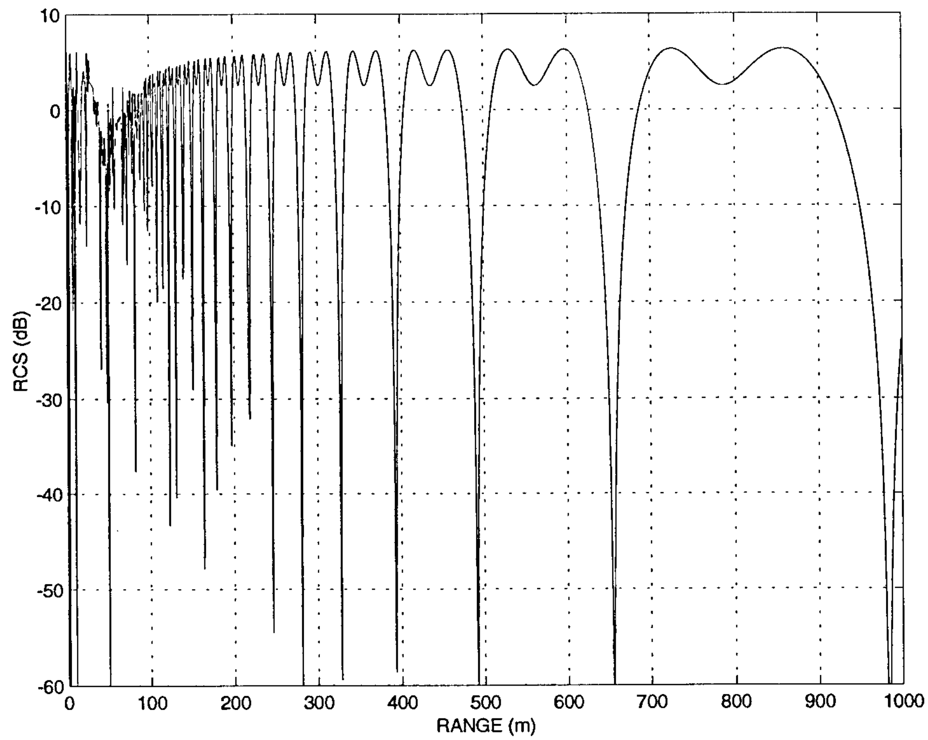

Fig. 7 — RCS with multipath, three reflectors at 0, 0.5, and 1 m perpendicularly above sea surface (radar height: 59 m; sea state: 0 (calm); $\rho = -1$)

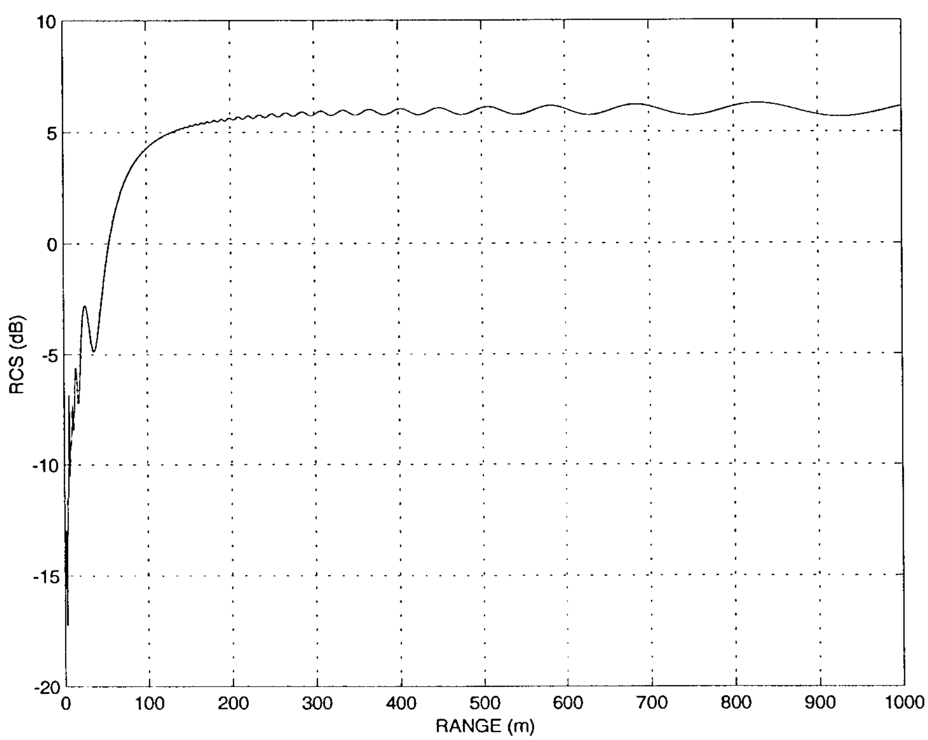

Fig. 8 — RCS with multipath, 667 reflectors perpendicularly above sea surface from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle $\alpha = 0$; sea state: 0 (calm); $\rho = -1$; and f = 10 GHz)

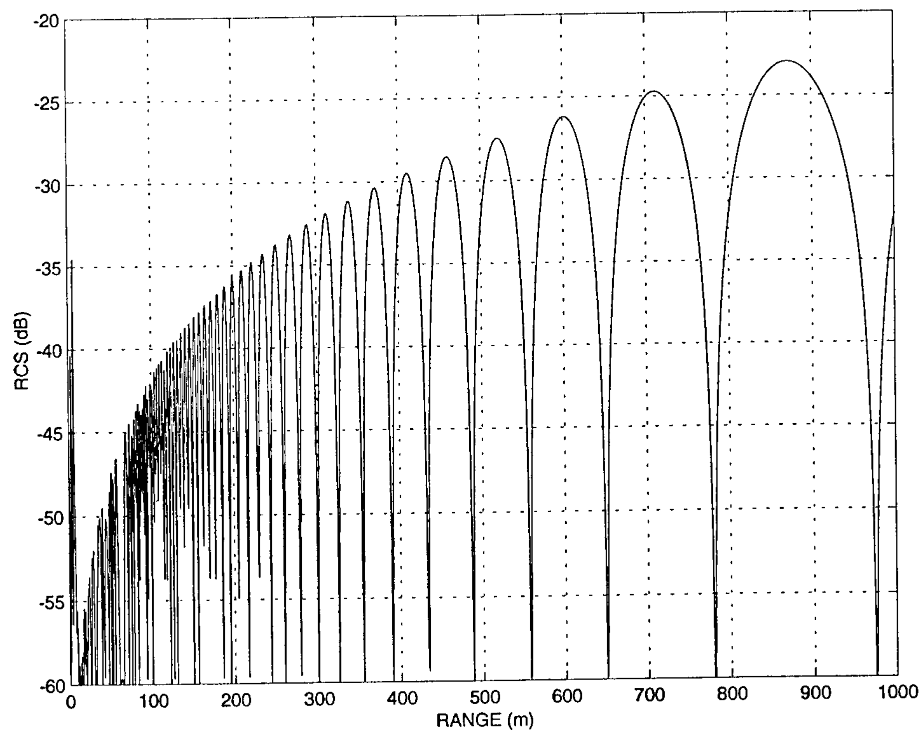

Fig. 9 — RCS with no multipath, 667 reflectors perpendicularly above sea surface from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle $\alpha = 0$; free space, no multipath; and f =10 GHz)

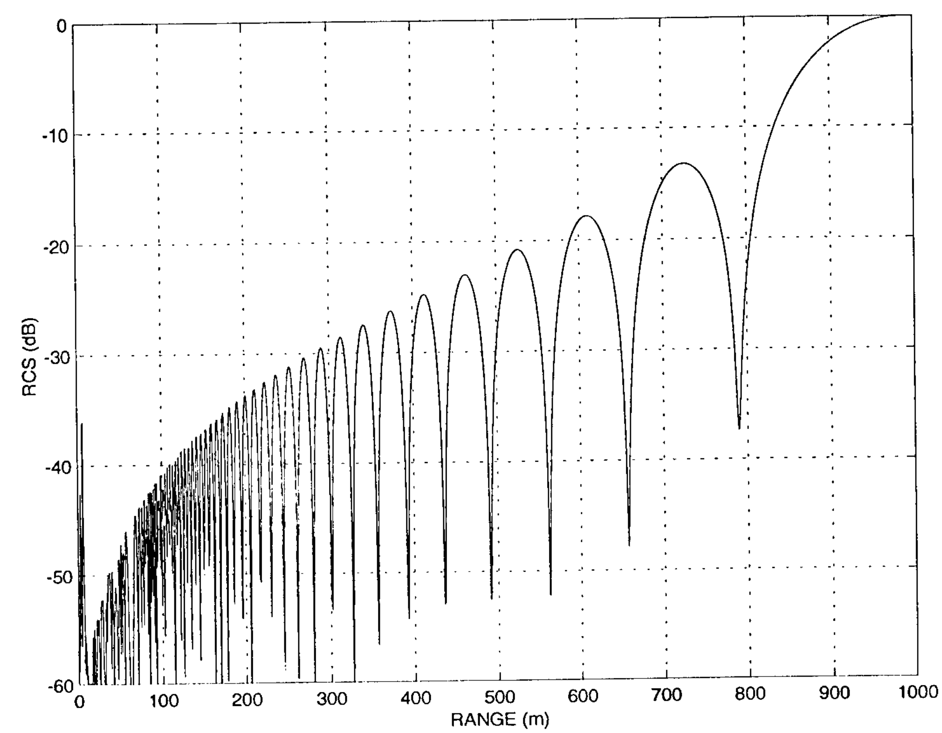

Fig. 10 — RCS with no multipath, 667 reflectors above sea surface, tilted 3.38° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle $\alpha = 3.38$; free space, no multipath; and f = 10 GHz)

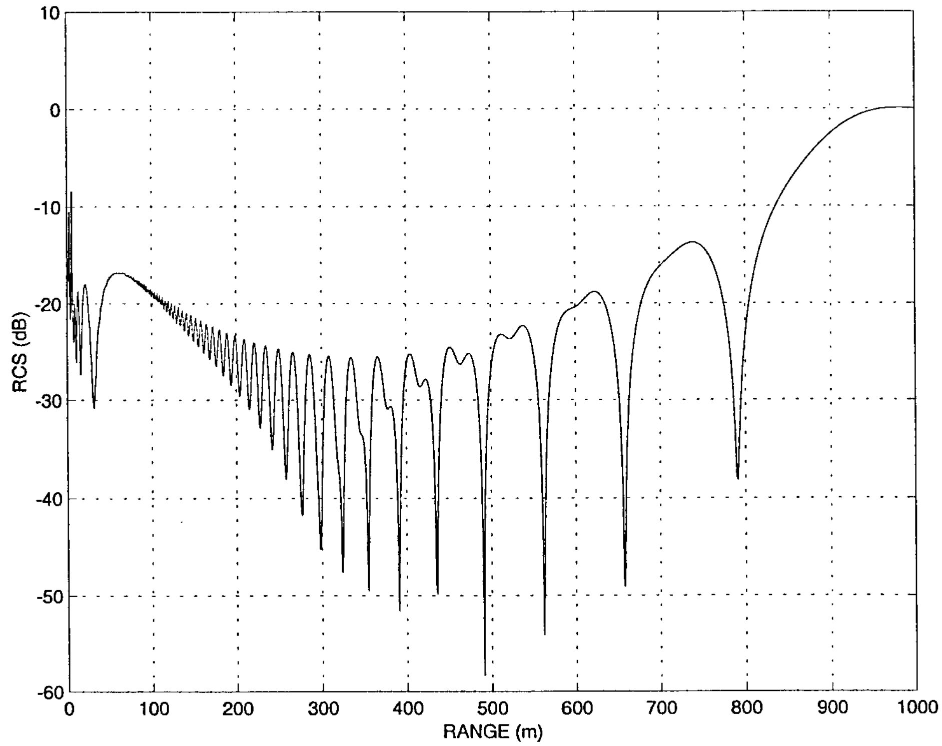

Fig. 11 — RCS with multipath, 667 reflectors above sea surface, tilted 3.38° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle α = 3.38°; sea state: 0 (calm); ρ = -1; and f = 10 GHz)

The angle of 3.38 degrees was interesting because it rendered the perpendicular aspect angle at 1000-m range. However, larger tilt angles such as 15 degrees may be of more interest.

Figure 12 shows the return of the reflector array tilted 15 degrees off perpendicular away from the radar, with multipath present. Again, at about 220 m range, the peak return of the flat plate pattern is seen, because at that point, according to the geometry, the radar beam hits the array perpendicularly. Remarkably, there is no enhancement of this peak due to multipath, because for that geometry, the main lobe of the reflection pattern of a flat plate does not hit the sea surface. With increasing height of the radar, this peak moves out farther in range.

Figure 13 shows the same RCS as in Fig. 12, but with no multipath. It is seen that at very close range like at 100 m, the RCS is enhanced by multipath by about 8 dB, at 900 m by about 5 dB. But, what happens at ranges farther out?

Figure 14 is like Fig. 13, but for a range out to 8000 m. The pattern seen is not due to multipath but rather to the varying aspect angle of the array. Figure 15 is like Fig. 14, but with no multipath reflections. It is shown that the peaks are increased by 5 to 6 dB due to multipath reflections. Figure 16 shows the pattern for a reflection coefficient of -10 dB (0.32). Hardly any difference is seen between no multipath and a -10 dB reflection coefficient.

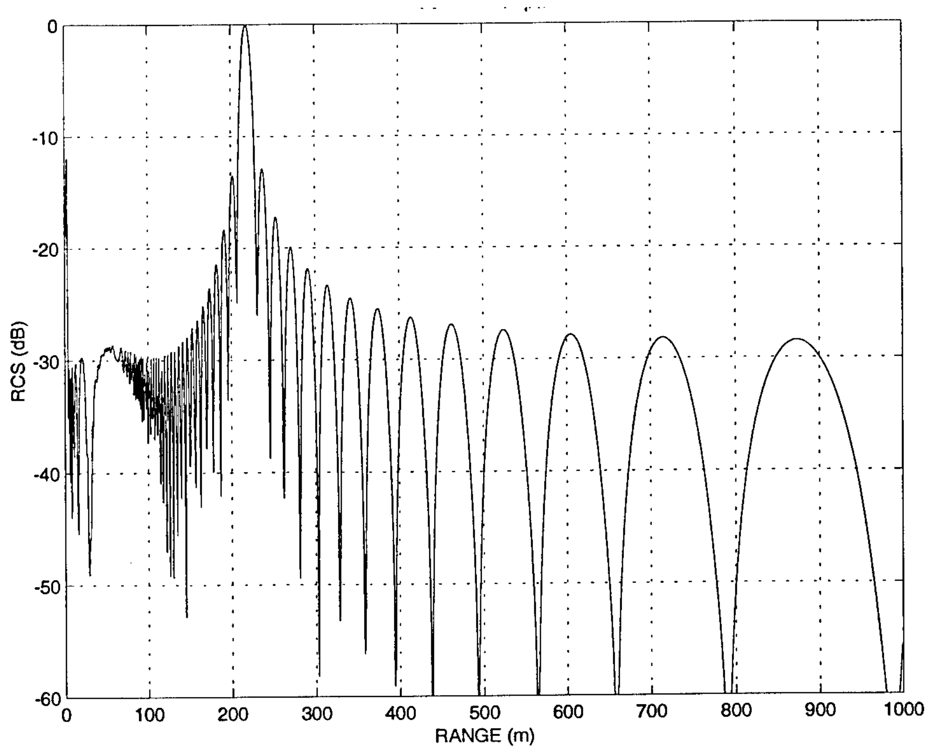

Fig. 12 — RCS with multipath, 667 reflectors above sea surface, tilted 15° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle α = 15°; sea state: 0 (calm); ρ = -1; and f = 10 GHz)

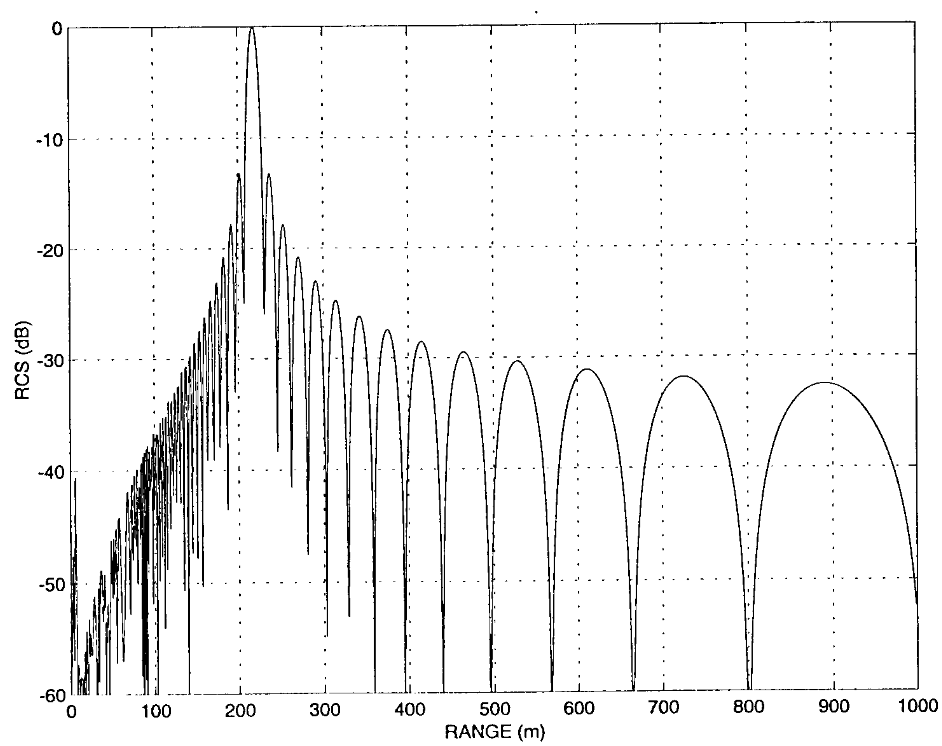

Fig. 13 — RCS with no multipath, 667 reflectors above sea surface, tilted 15° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle α = 15°; free space, no multipath; and f = 10 GHz)

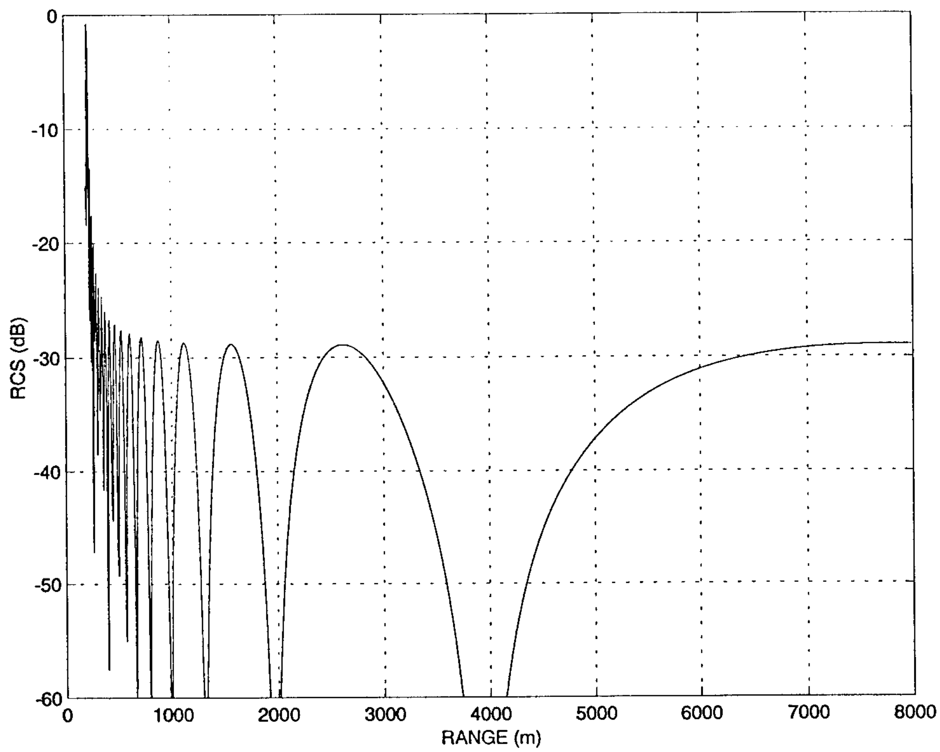

Fig. 14 — RCS with multipath, 667 reflectors above sea surface, tilted 15° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle α = 15°; sea state: 0 (calm); ρ = -1; and f = 10 GHz)

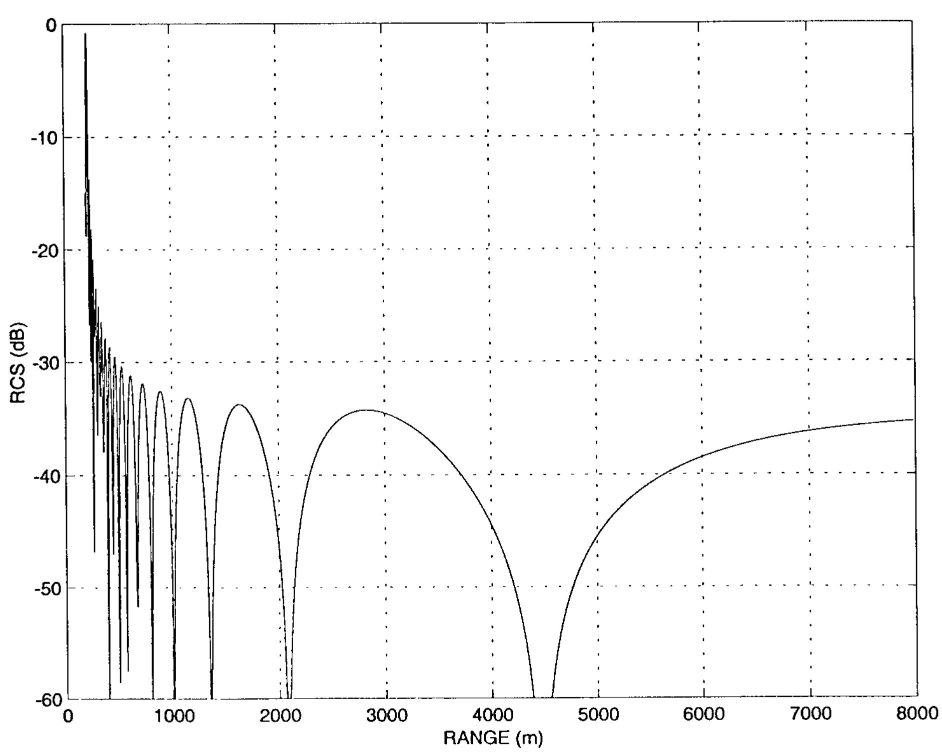

Fig. 15 — RCS with no multipath, 667 reflectors above sea surface, tilted 15° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle α = 15°; no multipath, free space; ρ = 0; and f = 10 GHz)

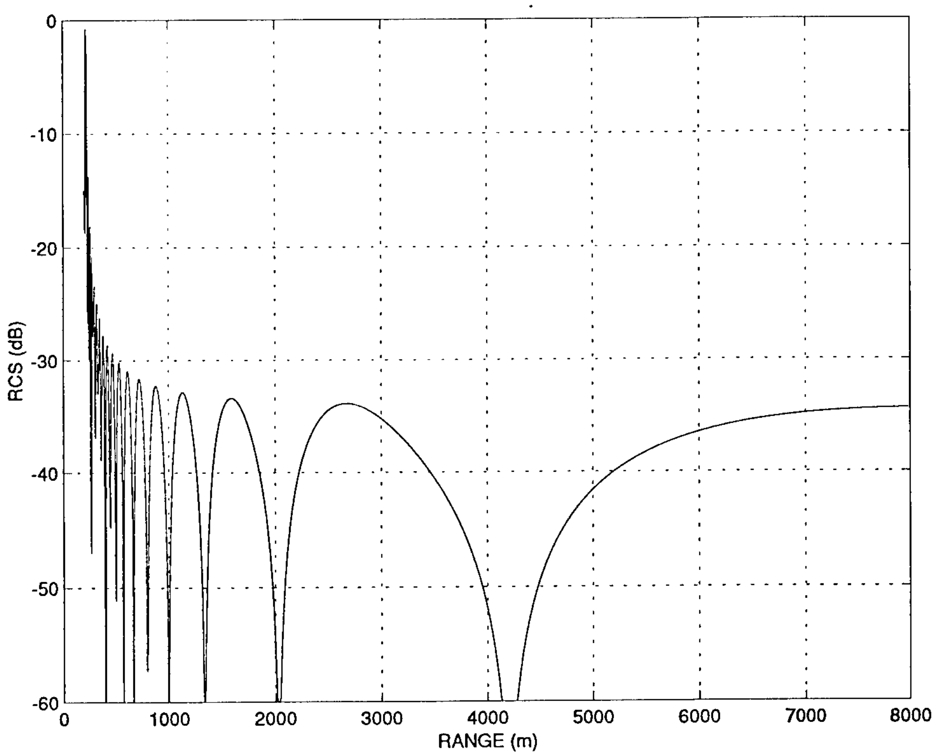

Fig. 16 — RCS with multipath, 667 reflectors above sea surface, tilted 15° from perpendicular, from 0 to 1 m (xmax = 1 m; xmin = 0 m; H = 59 m; tilt angle α = 15°; with little multipath; ρ = -0.32; and f = 10 GHz)

Distributed Reflector with Rough Surface

The influence of the roughness of the reflector is considered next. The reflector is made up of an array of small reflectors located along a line from the sea surface up to a height of 1 m, the line being tilted off the vertical by the angle α. Now, every adjacent 10 reflectors are moved out of the line towards the radar by delta_r, delta_r being equally distributed between zero and $\lambda/4$. Every 10 adjacent reflectors had the same delta_r. There were 20 reflectors per wavelength.

Figures 17 and 18 show the effect of reflector roughness for a smooth sea (ρ = -1) and for free space (ρ = 0), respectively. Figures 19 and 20 show the same for the range extended to 8 km. It is remarkable that the peak at the range where the radar beam hits the reflectors perpendicularly is very little degraded from its original value of 0 dB. For ranges greater than 1000 m, the RCS with full multipath is greater by 5 to 10 dB than the free space RCS. The maxima and the minima resulting from the reflection pattern of the smooth array are averaged out for the rough array at ranges above 1000 m, in that geometry.

CONCLUSION

If the surface of the reflector is rough (i.e., the rough surface peaks are smaller than $\lambda/4$), then the maxima and minima resulting from the sidelobes of the array reflection pattern are smoothed out as a function of range. The RCS of the array in full multipath condition is 5 to 10 dB larger than that of free space for ranges of 1000 m or greater, for the geometry considered.

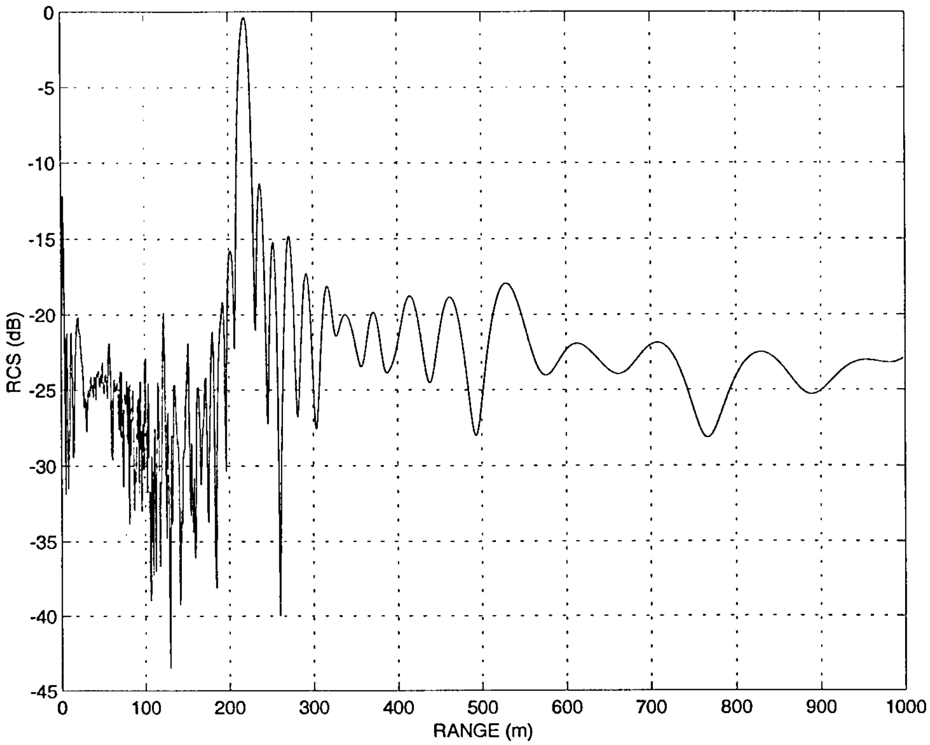

Fig. 17 — RCS with multipath, 667 reflectors above sea surface, tilted 15° from vertical, height from 0 to 1 m; H = 59 m; with multipath; ρ = -1; and f = 10 GHz

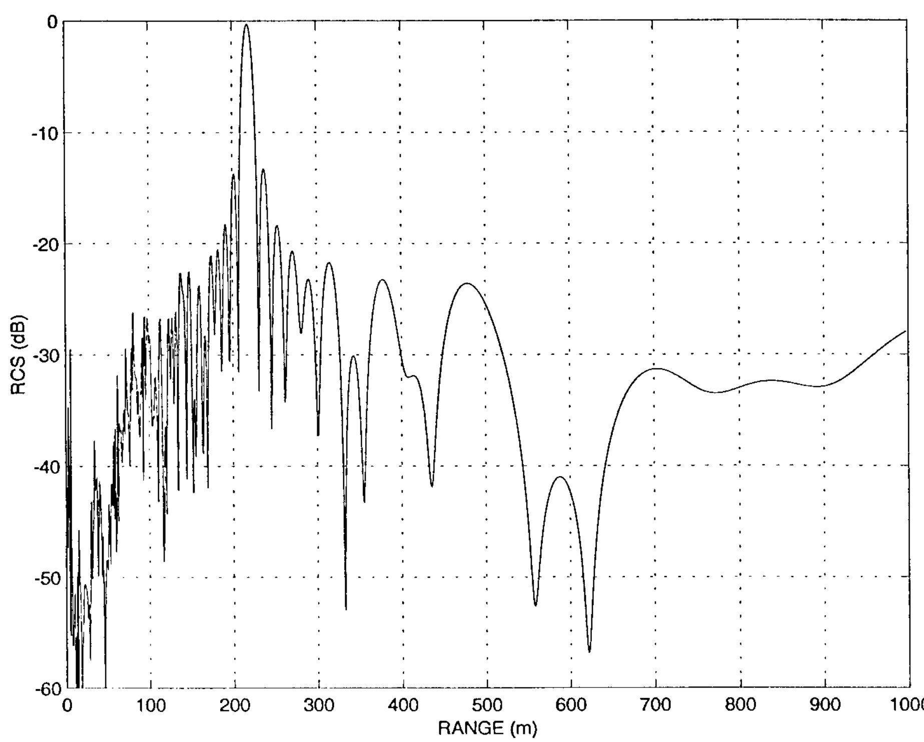

Fig. 18 — RCS without multipath, 667 reflectors above sea surface, simulating a rough reflector, tilted 15° from vertical, height from 0 to 1 m; H = 59 m; without multipath; ρ = 0; and f = 10 GHz

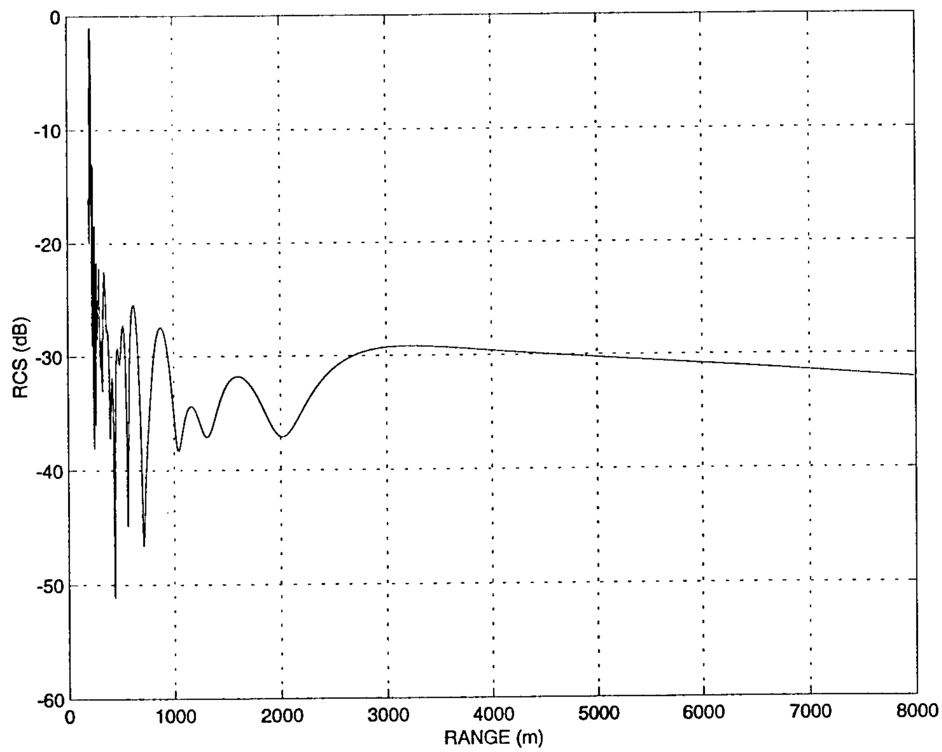

Fig. 19 — RCS with multipath, 667 reflectors above sea surface, simulating a rough reflector, tilted 15° from vertical, height from 0 to 1 m; H = 59 m; with multipath; ρ = -1; and f = 10 GHz

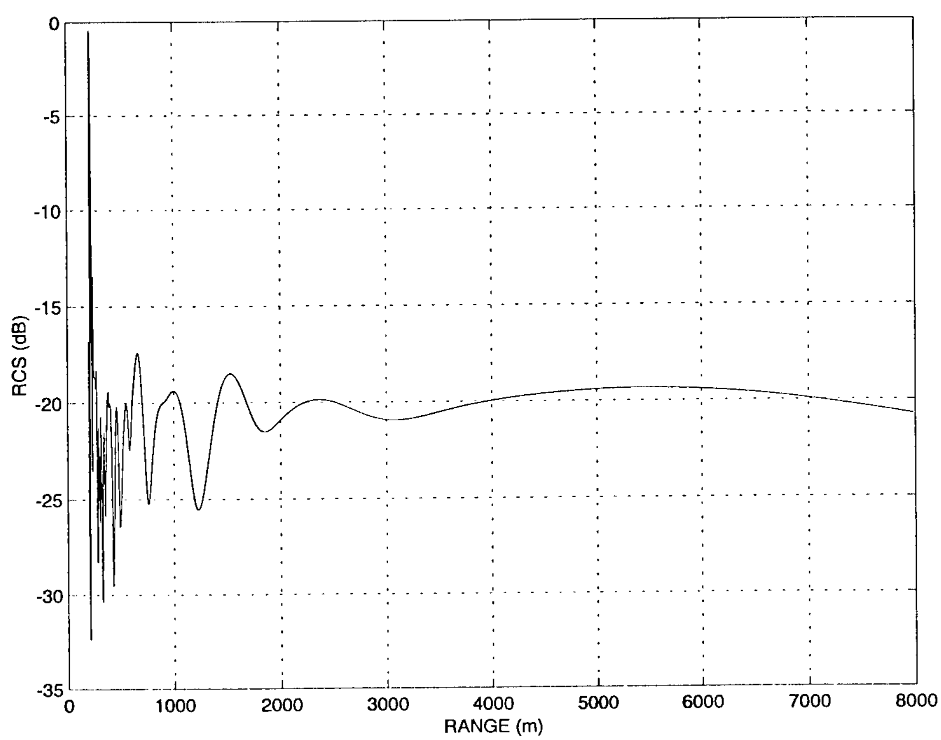

Fig. 20 — RCS without multipath, 667 reflectors above sea surface, simulating a rough reflector, tilted 15° from vertical, height from 0 to 1 m; H = 59 m; without multipath; ρ = 0; free space; and f = 10 GHz

With a reflection coefficient of -0.32 (-10 dB), there is practically no difference between free space and the multipath environment for this scenario. With smooth sea, sea state 0, reflection coefficient of 0 dB, the peaks in the 1-to-7-km ranges are enhanced by 5 to 6 dB. The peaks are not due to multipath but rather to the reflection pattern of a linear array as a function of aspect angle. As the range changes, so does the aspect angle.

Appendix
MATLAB PROGRAMS FOR FIGURES 2 THROUGH 4 (SPECRHO.M) AND 5 TO 16 (DISMULT.M)

```
% Program name specrho.m
% Calculates the specular reflection coefficient rho
% as a function of wave height, frequency and grazing angle as parameter
% According to Miller, Brown and Vegh,
% IEE Proceedings Vol 131 Pt. H No. 2 April 1984
% f = frequency (GHz)
% sigh = rms wave height (m)
% alf = grazing angle (degrees)
% rho = absolute value of specular reflection coefficient (dB)

f=3;
lam = .3/f;
alf=[0.1 0.2 0.5 1];
alfarc=alf*pi/180;
p=sin(alfarc)/lam;
delsig=0.001;
sigmin=0.1;
sigmax=10;
m=1;
while m<5

n=1;
sig=sigmin;

while sig<sigmax
   sig=sig+delsig;
   g=sig*p(m);
   u = 2*(2*pi*g)^2;
   if u>300
      u=300;
      end
      y= 20*(log10(besselj(0,j*u))-u*log10(exp(1)));
      if y<-30
         y=-30;
         end
   rhodb(m,n)= y;

sigh(n)=sig;

   n=n+1;
end
n=1;
sig=sigmin;
m=m+1
end
semilogx(sigh,rhodb(1,:),sigh,rhodb(2,:),sigh,rhodb(3,:),sigh, rhodb(4,:))
grid
```

```
% Radar response of distributed reflectors low over the sea surface
% with multipath reflections. The grazing angle is assumed to be small
% (smaller than  the Brewster angle). The Reflector is an assembly of
% vertically equally spaced equal reflectors, emulating a flat plate.
%  The whole assembly is tilted from the perpendicular by an angle
%  alfa of less than 20 degrees.
%  The reflection coefficient rho on the sea surface is equal to -1 for
% glassy smooth sea (sea state 0) for small grazing angles and horizontal
% polarization. The fourth power of range of the radar range equation is
% omitted in the result. The total RCS of the reflectors in free space
%  is normalized to 0 dBsm.
% Parameters:
%  lam        = Electrical wave length (m)
%  h          = Height of radar above the water surface
%  rho        = reflection coefficient of the water surface
%  xmax       = maximum height of distributed reflectors (m)
%              (the lowest reflector is assumed to be on the water surface)
%  alf        = tilt angle of the reflector assembly from perpendicular
%  nrefl      = number of reflectors (depends on lam and xmax)
%  r          =  range (m)
%  rmin       = minimum range
%  rmax       = maximum range
lam=0.03;
h=59;
%xmax=1;
xmax=1;
xmin=0;
%alf=2.78;
%alf=3.38;
alf=15;
%alf=0;
nrefl= 20*(xmax-xmin)/lam;
% The spacing of the reflectors is lambda/20
%nrefl=2;
%x=0:xmax/nrefl:xmax;
x=xmin:(xmax-xmin)/nrefl:xmax;
%x=1:1:1;
%x=xmax:xmax:xmax;
% rmax=8000;
% rmin=200;
rmax=1000;
rmin=1;
%rmin=670;
%x=0.3;

rho=-1;
% rho=0;
%rho=-0.316;
nmax=1000;
n=1;

alfarc=alf/180*pi;
talf=tan(alfarc);
while n<nmax
   r=rmin+(n-1)/nmax*(rmax-rmin);

   ar(n)=r;
   k=2*pi/lam;
   phi=2*k*x*talf;
   aa=k/r*(x+h).^2 + phi;
   bb=k/r*(x-h).^2 + phi;
   cc=k/r*(x.^2+h^2) + phi;
   v(n)=abs(sum(exp(j*bb)+2*rho*exp(j*cc)+rho^2*exp(j*aa)))/length(x);
   if v(n) < 1e-3
      v(n)=1e-3;
      end
      lv(n)=20*log10(v(n));
```